

## AN ADAPTIVE SPECTRAL RESPONSE MODELING PROCEDURE FOR MULTI-PORT WAVEGUIDE JUNCTIONS

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### Abstract

An adaptive scheme is proposed to generate the spectral response of waveguide junctions in minimum computation time. The procedure uses the newly developed transfinite element method to determine the fields in junctions at a few adaptively selected frequencies and then employs these solutions to generate the spectral response over the full frequency range.

### Introduction

N-port waveguide junctions are solved numerically in the literature by using one of two approaches: the eigensolution method<sup>1,2</sup> and the deterministic method<sup>3,4</sup>. While the eigensolution method is mathematically elegant, it has the disadvantage of requiring the solution of large matrix eigenvalue equations. For this reason, recent work has focused on the deterministic approach in which the response of the system is computed at only one frequency<sup>3,4</sup>. In the existing work, however, the formulation results in either a non-symmetric matrix equation<sup>3</sup> or a set of boundary value problems<sup>4</sup> that must be solved separately for each frequency of interest.

In this paper, we introduce a new, highly efficient procedure for modeling N-port waveguide junctions. The basis of the procedure is the transfinite element method<sup>5</sup> in which analytical procedures are combined with finite element basis functions to provide solutions for problems involving open boundaries. This procedure

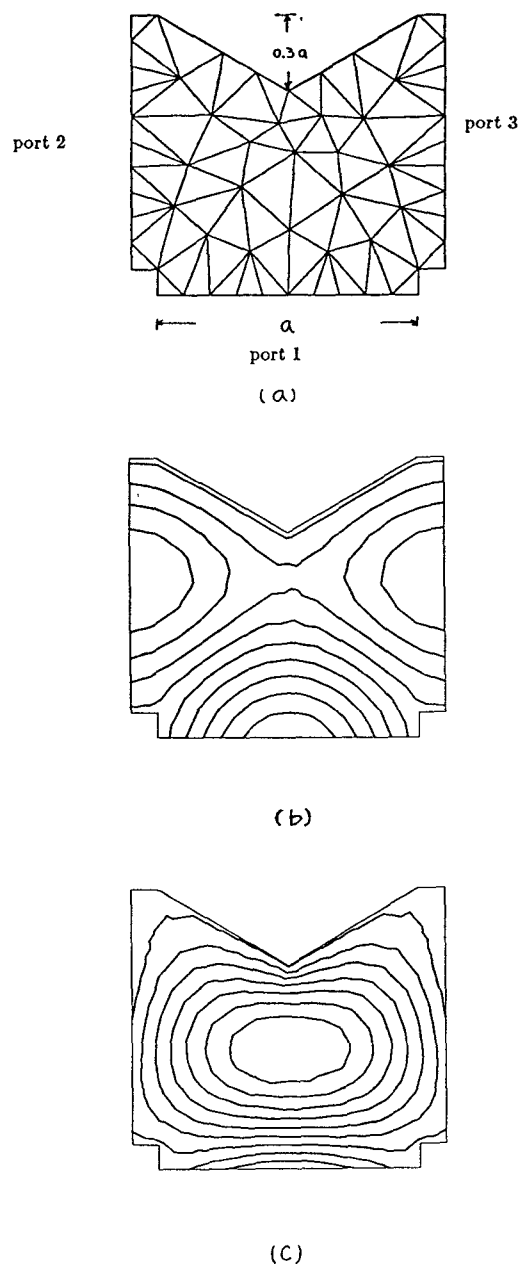
results in a symmetric sparse matrix equations that can be solved very efficiently using the pre-conditioned conjugate gradient algorithm. Further, we develop here a spectral response estimation procedure by which solutions at a few adaptively selected frequencies are used to generate the solution in the entire frequency range of interest. With large problems, this approach is orders of magnitude more efficient than the existing alternatives.

### Transfinite Elements

In the transfinite element approach<sup>5</sup>, the analysis in general involves into three types of models: (1) a finite element model over a finite-sized region, (2) analytical solutions over infinite regions, and (3) a singularity model for small circular regions around singular points. By determining the stationary points of a bilinear functional corresponding to the Helmholtz equation, a symmetric matrix equation is produced. The time required to solve this equation by the pre-conditioned conjugate gradient algorithm is  $\text{Order}(n^{1.3})$  where  $n$  is the matrix size. Figure 1 presents the finite element mesh and the real and imaginary electric fields produced by using this procedure for a three-port junction.

### Adaptive Spectrum Analysis

In designing microwave circuits, the frequency response over a given frequency range is often required. With the deterministic approach<sup>3,4</sup>, the frequency response must be computed many times to give sufficient accuracy everywhere. In the adaptive approach developed here, the matrix equation approximating the system is solved at only a few selected frequencies and then these solutions are used as basis functions to generate the full spectral response.



**Figure 1.** Transfinite element analysis of a T-junction. (a) Finite element mesh. (b) Plot of the real component of equal E contours at  $ka/\pi = 1.5$ . (c) Plot of the imaginary component.

The adaptive approach to spectral response modeling is outlined in Figure 2. In Figure 2(a), the three-port junction of Figure 1 is solved by using the transfinite

element method at the two limiting frequencies indicated by the squares on the  $k$ -axis. These two solutions are then used as basis functions to generate a crude spectral response curve throughout the region of interest. This is plotted as solid lines in Figure 2(a). Next, we compute the error in the solution throughout the frequency range by substituting the crude solution values into the governing equations for the system and evaluating the residual. This is done very efficiently. We then solve the system once again at the frequency that gave the maximum residual on the last pass. The new square in Figure 2(b) shows the location of this solution as well as the new spectral response curve computed by using the three computed solutions as basis functions. This procedure is repeated for five iterations in Figure 2 until the error in the entire spectral response curve is within acceptable limits. As is evident from Figure 2(e), the procedure converges to the solution given in reference 3. In this case, the adaptive procedure required less than 1% of the computation time required by the direct deterministic procedure.

## References

1. P. Silvester, "Finite Element Analysis of Planar Microwave Networks", *IEEE Trans. Microwave Theory Tech.*, vol MTT-21, FEBRUARY 1973, pp. 104 - 108.
2. G. D'Inzeo, F. Giannini, C. M. Sodi, and R. Sorrentino, "Method of Analysis and Filtering Properties of Microwave Planar Networks", *IEEE Trans. Microwave Theory Tech.*, vol MTT-26, JULY 1978, pp. 462 - 471.
3. M. Koshiba and M. Suzuki, "Application of the Boundary-Element Method to Waveguide Discontinuities", *IEEE Trans. Microwave Tech.*, vol MTT-34, FEBRUARY 1986, pp. 301 - 307.
4. J. P. Webb and S. Parihar, "Finite element analysis of H-plane rectangular waveguide problems", *IEE Proceedings*, vol 133, Pt. H, April 1986, pp. 91 - 94.
5. J. F. Lee and Z. J. Cendes, "Transfinite Elements - A Highly Efficient Procedure for modeling Open Field Problems", *Journal of Applied Physics*, accepted for publication, .

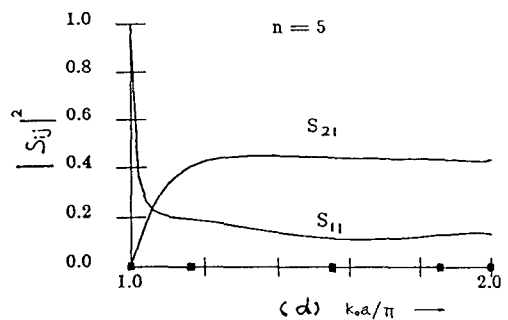
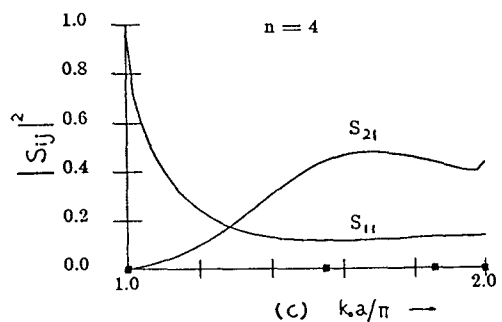
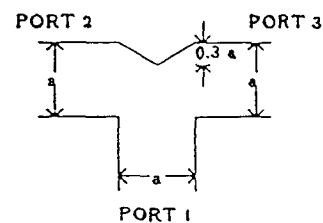
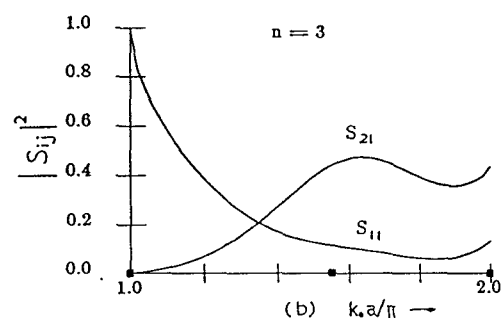
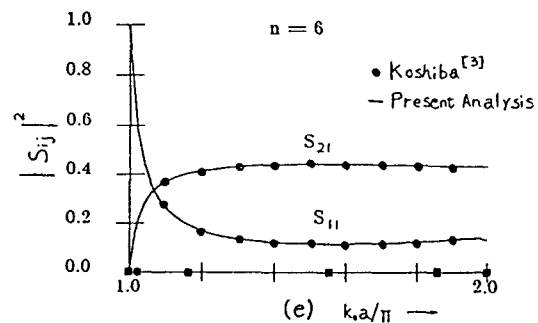
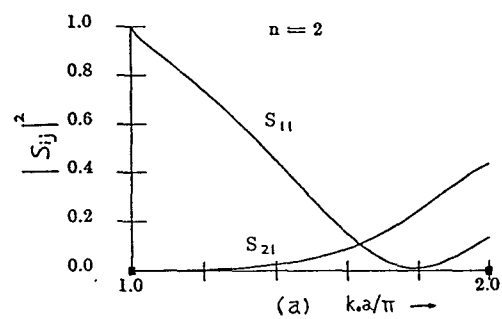


Figure 2. Adaptively computed reflection and transmission coefficients of the T-junction in Figure 1.